Spring washer effect description

Paper by Graeme Everett

Background

The CIG has concluded that the prices for period 36 were the result of a so-called spring washer effect. In the Tauranga case the CIG has explained the spring washer as follows:

1. A transmission constraint existed.
2. The constrained circuits were part of a loop.
3. Kirchhoff's Law applies around all loops.
4. This has caused a spring washer.
5. Generation was re-arranged to meet the demand at Tauranga in such a way as to cost the system $8,272 for a small increment of demand at Tauranga.

The CIG has considered if this analysis provides an adequate explanation of how the spring washer worked, and why the price at Tauranga was $8,272/MWh. It has been concluded that whilst the analysis has provided a good description of what occurred, it provides no understanding as to why the situation occurred.

One member of he CIG has provided a short paper that uses a simple 3-node grid to explain the principles of how spring washers work.

Spring washer

Kirchhoff's Law

Kirchhoff's Law has been represented in SPD by the following equation:

$$P_{ij} = \frac{(\theta_i - \theta j)}{x_{ij}}$$

where P_{ij} is the power flow between two nodes i and j, X_{ij} is the reactance of the physical component linking i and j (typically either an AC transmission circuit, a series reactor or a transformer) and θ_i is the voltage angle at node i.

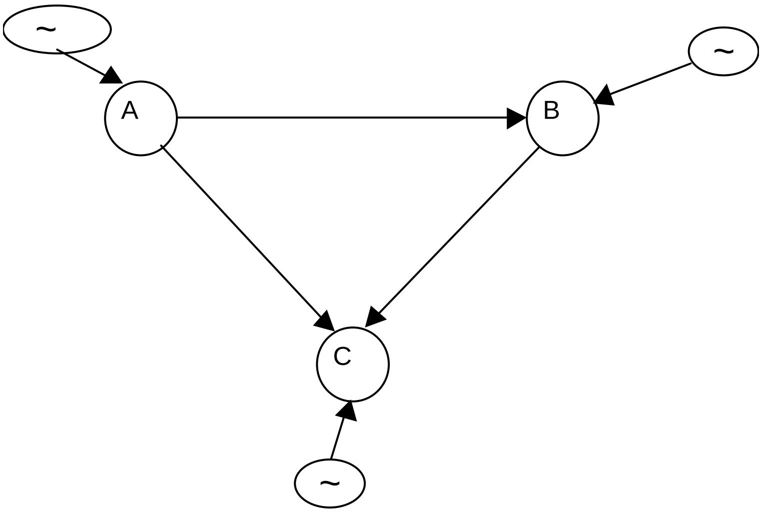
This equation ensures that Kirchhoff's Law is respected at every node in the network.

An alternative representation is as follows:

$$\sum_{ij} P_{ij} X_{ij} = 0$$ for all i j in a loop.

In other words the sum of the power flow multiplied by reactance around a loop must be zero. This representation would be clumsy within the SPD model as it requires manual identification of each and every loop. But it is helpful for the purpose of explaining spring washers.

It is useful to consider a simple example in order to understand how this equation works, and how it leads to a spring washer. Consider a simple loop involving 3 nodes, as follows:



Note that node C draws load. Generators can inject at nodes A and B. An artificial generator with a very high cost has been attached to node C (for the purpose of avoiding an infeasible solution when the load flow problem for this simple network is solved with a linear program).

The circuits linking nodes A, B and C have the following characteristics:

	Reactance	Loss	Limit
A->B	0.012	0%	100
B->C	1	0%	100
A->C	0.001	0%	100

Note that losses of zero % means that losses have been ignored.

Once off-take information and generation offer data have been added to this network, a simple load-flow problem can be solved, and nodal prices can be determined for nodes A, B and C.

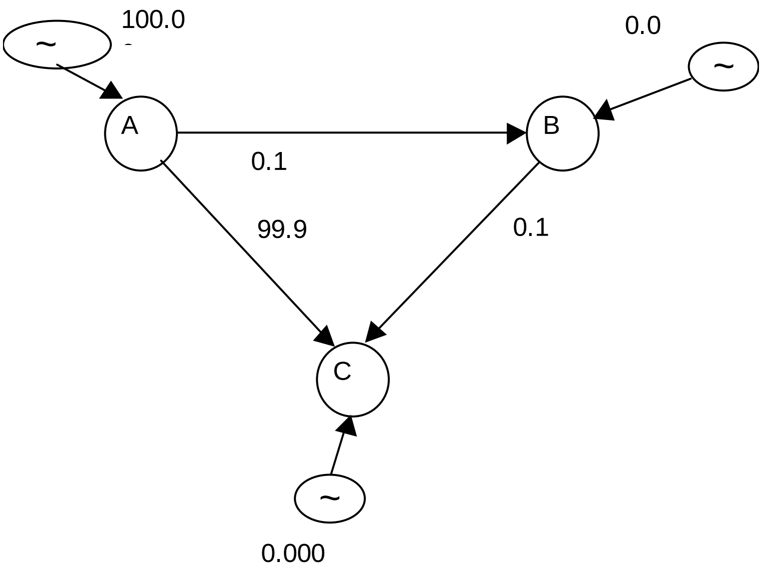
Note that for all examples load is only applied to node C.

Example 1

	Node A	Node B	Node C
Generation Offer	200	200	100
Price	$100	$200	$100,000
Load	0	0	100

Here the generator at node A offers 200MW at $100/MWh, the generator at node B offers 200MW at $200/MWh and the artificial generator at node C offers 100MW at $100,000/MWh.

The solution is as follows:



The nodal prices are:

	Nodal Price
A	$100
B	$100
C	$100

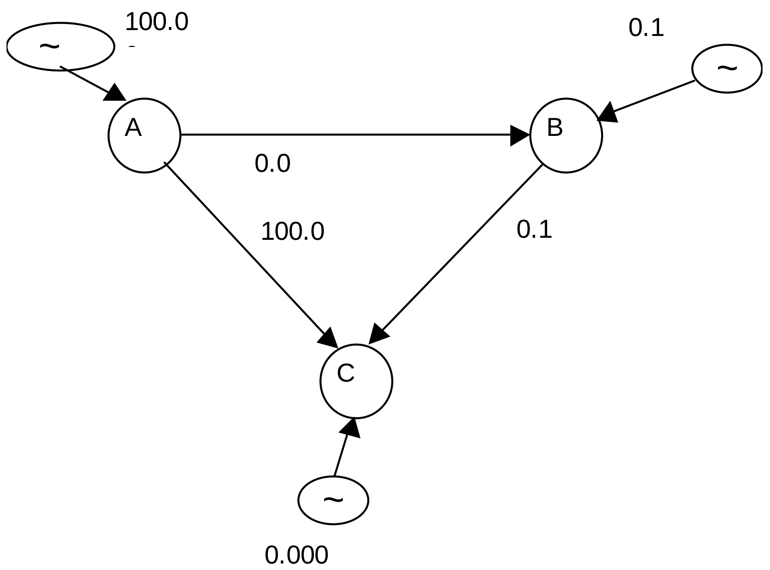
The loop constraint:

$P_{AB}X_{AB} + P_{BC}X_{BC} + P_{CA}X_{CA} = 0$ is respected as follows:

$0.1 \times 0.012 + 0.1 \times 1 - 99.9 \times 0.001 = 0$

Example 2

If the load at node C is increased to 100.1MW, and all other parameters are kept the same as in experiment 1 the following solution is obtained:

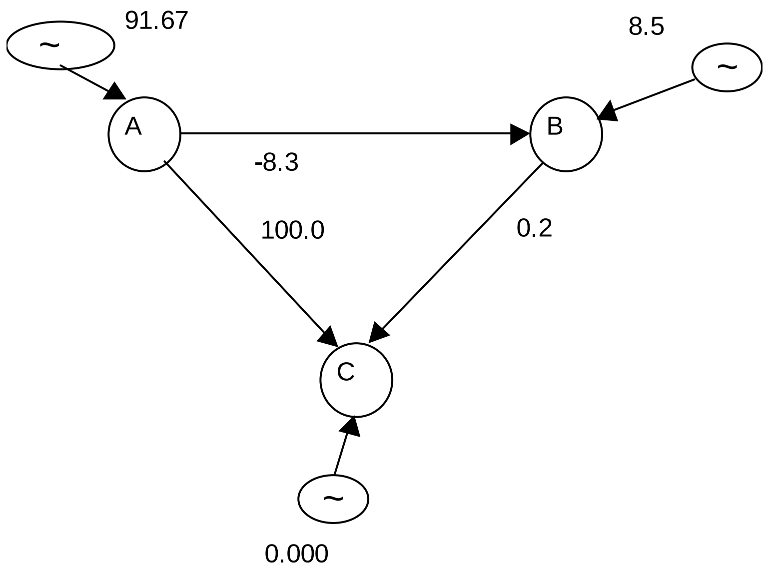


	Nodal Price
A	$100
B	$200
C	$8,533

The generator at node B is required to generate 0.1MW. As the generator at node B has an offer price of $200/MWh this sets the nodal price at node B. The nodal price at node C of $8,533 arises due to the effect of the loop constraint. This is not immediately obvious, and requires a further experiment to be explained.

Experiment 3

If the load at node C is increased to 100.2MW, and all other parameters are kept the same as in experiment 1 the following solution is obtained:



In order for the loop constraint to be respected, the generator at B is required to inject 8.5MW. Only 0.2MW flows directly between B and C, the other 8.3MW flows from B to A and on to C. The generator at A is therefore forced to reduce generation to 91.67MW.

The loop constraint:

$P_{AB}X_{AB} + P_{BC}X_{BC}+P_{CA}X_{CA} = 0$ is respected as follows:

-8.3 x 0.012 + 0.2 x 1 –100.0 x 0.001 = 0

Explanation of node C nodal price

The same basis matrix was observed at optimality in experiments 2 and 3. Therefore the following calculation is valid.

The difference in dispatch of the generator at node B between experiments 2 and 3 is 8.4MW.
The difference in dispatch of the generator at node A between experiments 2 and 3 is –8.33MW.
The difference in demand between experiments 2 and 3 is 0.1MW.

It can be readily observed that increasing the demand at node C by 0.1MW causes 8.4 extra MW to be generated at B, and 8.33 fewer MW generated at A. The price at node C can be observed to be the difference in the cost of dispatch between experiments 2 and 3:

(8.4 x \$200 – 8.33 x \$100)/0.1 = \$8533/MWh

Effect of loop constraint

The reason the dispatch has changed is to respect the loop constraint. The circuit linking nodes A and C has a very low reactance value compared with the circuits linking A and B, and B and C. This has the effect of pushing as much flow as possible between A and C. However once the flow in the circuit between A and C has reached its upper bound of 100MW (in other words it is 'constrained') the only other way to send power to node C is by sending 0.2MW from B to C (ignoring the artificial generator at node C).

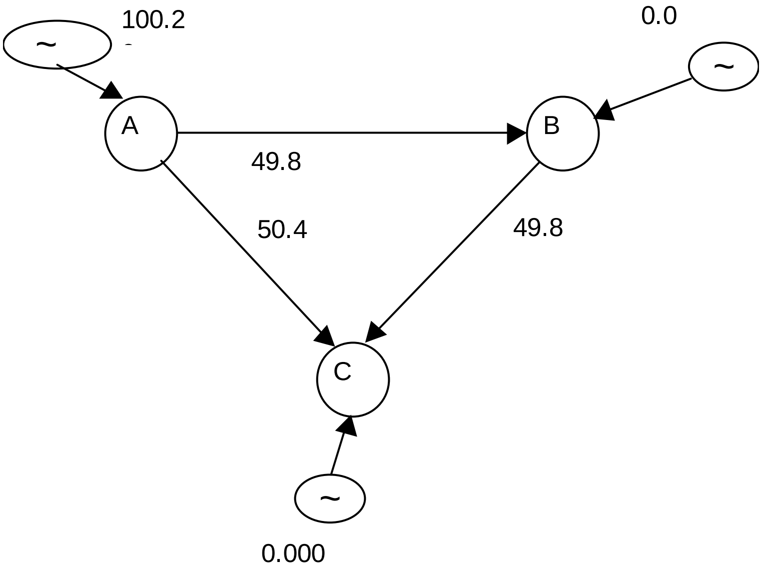
It follows that if 100MW flows from A to C, and 0.2MW flows from B to C, then there must be flow back from B to A that satisfies the loop constraint. This flow is 8.3MW. Note that in this case the ratio of reactance in the circuits B to C, and A to B determine the price at node C, viz:

Price at node C approximately equals X_{BC}/X_{AB} x Price at A

The small increase of demand at node C has resulted in a relatively large change in the dispatch of generation and flows along the transmission circuits, as well as a high nodal price at C. In the New Zealand electricity industry this situation is often described as a ’**spring washer**’.

Experiment 4

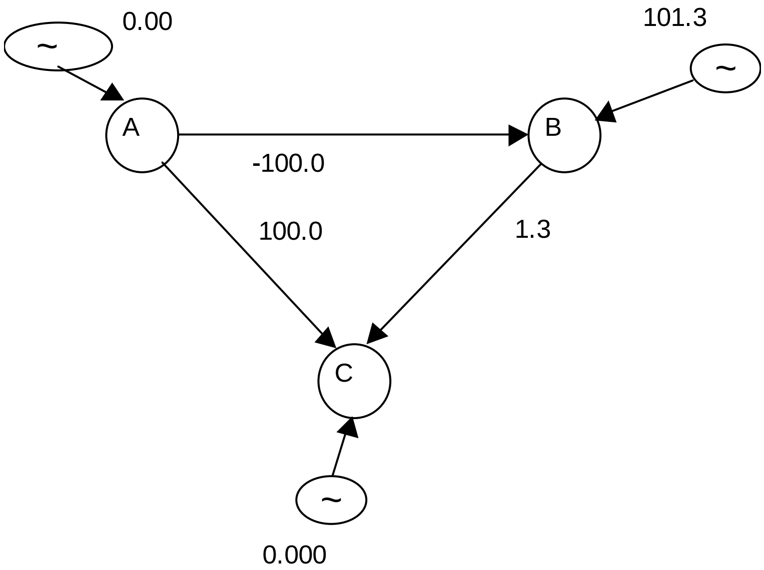
It is interesting to observe that if all other parameters remain the same as experiment 3, but the reactance of the line between A and C is increased to 1, then a completely different solution is obtained as follows:



	Nodal Price
A	$100
B	$100
C	$100

Experiment 5

If the load at node C is increased to 101.3MW, and all other parameters are kept the same as experiment1 then the following solution is obtained:



	Nodal Price
A	-$998
B	$200
C	$100,000

It can be observed that although the artificial generator at node C is not dispatched, the prices are consistent with the offer price of $100,000 at C. This is a common outcome when the optimal solution to the linear program is degenerate, which is the case here.

Note that any increase in load at C beyond 101.3MW is met by the artificial generator at C.

Conclusions

From these experiments the following conclusions can be drawn:

1. The ratio of reactance values in circuits comprising a loop has a major influence on the transmission flows.
2. Loop constraints have no material effect on nodal prices unless power flows along one of the transmission circuits in the loop has reached its upper limit.

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